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Bending calculation of multi-span beam under arbitrary boundary conditions and engineering application thereof

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Abstract: [**Objectives**] To find the worst-case of a deck structure under patch loading quickly. [**Methods**] For the bending of a multi-span beam under arbitrary boundary conditions, an Improved Fourier Series Method (IFSM) is used to describe the displacement functions of the multi-span beam, list the boundary equations that the displacement functions need to meet and solve such equations to obtain the relational expressions of coefficients; and then an energy control equation is obtained on the basis of the Hamilton principle, the displacement functions of beam structure satisfying the boundary conditions is acquired with the Galerkin method, and the functions are compared with the finite element results by means of example analysis. Finally, this method is applied to the calculation of the worst-case analysis of multi-span beam under patch loading. [**Results**] The results show that the error between the result of this paper and the finite element analysis is less than 0.05%, indicating good accuracy. [**Conclusions**] Compared with finite element method, the speed of solving the worst-case of the multi-span beam is greatly reduced by using this method, and a more accurate location to which the worst-case of patching loading is applied is obtained by combining the genetic algorithms.

Key words: multi-span beam; patch loading; bending; arbitrary boundary conditions; Hamilton principle; worst-case analysis

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0 Introduction

The multi-span beam model is a common mechanical model in ship and bridge fields. At present, there are many studies all over the world on the vibration and impact of beam structures under arbitrary boundary conditions. For example, References [1-3] have proposed an improved Fourier series method (IFSM), which adds four sine series to the traditional Fourier cosine series, namely a semi-analytical method (SAM). It is proved by mathematics that this method can extend and converge to any function. The correctness of this method is verified by the vibration analysis of beams and decks with elastic boundary conditions. Xu and Li [4] added four polynomial series to the traditional Fourier cosine series (the method can also eliminate the discontinuity of Fourier series on the boundary) and studied the response of multi-span beams under dynamic load. In addition, Zhou and Shi [5] also used the series form to

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study the vibration of multi-span beams under arbitrary boundary conditions. At present, the bending problem of beams or multi-span beams under arbitrary boundary conditions is mainly solved by the classical structural mechanics method or finite element method (FEM). The former needs to solve the three-moment equation or the five-moment equation first and then obtain the displacement of each span from the solved node moment. Finally, the result of the physical quantity to be solved is obtained by the differential relationship between other physical quantities and the displacement. In terms of the FEM, the model needs to be established in the pre-processing first and then solved. Therefore, the whole process takes a long time, and it is difficult to modify when the geometric parameters change.

In Ro-Ro ships and large ships, the main load-bearing members such as web beam and longitudinal girder on vehicle loading deck and hangar deck can be regarded as a kind of multi-span beams. Under the patch loading, the load size and action location are uncertain. Therefore, it is of great significance to find the worst-case of multi-span beams under various patch loading conditions for the safety check of ship structure and the design of the loading scheme. Jeon and Kim[6] studied the performance of the genetic algorithm applied to the worst-case analysis and successfully found the worst-case of multiple classical mathematical problems. Fang et al. [7] simplified the patch loading into the concentrated force to make an experimental device for continuous multi-span beams model under patch loading and compared the theoretical and experimental values of bending moment. The results showed that they were basically consistent. Kang et al.[8] proposed a method combining the FEM and the genetic algorithm. First, they calculated the response value of multi-span beams under various load conditions by the FEM and then adopted the maximum bending moment or maximum shear force of each span as the objective function to obtain the worst-case of each span under the patch loading conditions by the genetic algorithm, so as to carry out the optimization design of multi-span beams. However, the FEM consumes a long time while the traditional genetic algorithm requires a lot of calculation design points, leading to low overall calculation efficiency. Besides, because the location of patch loading adopted the discrete value, it is impossible to find a more accurate worst-case of patch loading.

In view of this, the paper first deduces the equilibrium equation of bending problems of multi-span beams under arbitrary boundary conditions by IFSM and based on the Hamilton principle. Then, the equilibrium equation is combined with the boundary conditions to obtain the solution, which is verified by comparing it with the FEM results. Finally, the method is further combined with the genetic algorithm for continuous variable optimization to solve the worst-case of multi-span beams when the location of patch loading is a continuous variable, so as to obtain more accurate working conditions.

1 Theoretical bending calculation of continuous multi-span beams

1.1 Physical model

As shown in Fig. 1, it is assumed that the physical parameters of the i-th span of n-span continuous beams are l_i, E_i, and I_i, which respectively refer to the span length, elastic modulus, and section inertia moment of the i-th span. For the stiffness coefficient of the spring on the boundary, it is stipulated as follows: k_0 and K_0 refer to the stiffness coefficient of the displacement spring and torsion spring at the head end of the continuous beam; k_n and K_n refer to the stiffness coefficient of the displacement spring and torsion spring at the tail end of the continuous beam; k_i refers to the stiffness coefficient of the displacement spring of intermediate support, on which an arbitrary form of transverse bending load $q(x)$ is applied. The arbitrary boundary condition can be simulated by changing the stiffness coefficient of each spring. For example, when the model is rigidly fixed at both ends and simply supported in the middle, it is enough to set the spring stiffness coefficients (k_0, K_0, k_n, K_n) of both ends and the spring stiffness coefficient k_i of the intermediate support as infinite. For simulating the free boundary, we only need to set the stiffness coefficient of each spring to be 0.

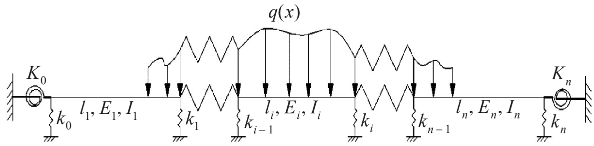


Fig.1 Schematic diagram of the multi-span continuous beams model under arbitrary boundary conditions

1.2 Semi-analytical solution of the model

1.2.1 Expression of displacement series

For the i-th span displacement function of continuous beams under arbitrary boundary, it can be expressed by the following IFSM [2]:

$$w_i = \sum_{m=-4}^{\infty} A_{im}\varphi_{im}(x_i) \quad (1)$$

where

$$\varphi_{im}(x_i) = \begin{cases}\cos\lambda_{im}x_i, m \geqslant 0\\ \sin\lambda_{im}x_i, m < 0\end{cases}, \quad \lambda_{im} = \frac{m\pi}{l_i} \quad (2)$$

where w_i is the i–th span displacement of the continuous beam; φ_{im} is the shape function, and the subscript m is the integer number of each series; A_{im} is the coefficient of the shape function; x_i is the position coordinate of a point within the i–th span of the continuous beam; l_i is the length of the i–th span of the continuous beam.

In addition to the traditional form of Fourier cosine series, the above displacement series adds four terms of Fourier sine series, overcoming the discontinuity or jump of a single–form Fourier series at the boundary. Mathematically, the series form can extend and converge to any function $f(x)$. In the actual calculation, the upper limit of the displacement series is a positive integer M, namely the truncation number of the series. Each displacement w_i contains $(M + 5)$ unknown coefficients A_{im}, and solving the $n \times (M + 5)$ unknown coefficients is the key.

1.2.2 Processing of boundary conditions

For the n–span continuous beams shown in Fig. 1, the boundary equation at the head end can be expressed by Eqs. (3) and (4) respectively, and that at the tail end can be expressed by Eqs. (5) and (6) respectively:

$$k_0 w_1 = -E_1 I_1 w_1''' \quad (3)$$

$$K_0 w_1' = E_1 I_1 w_1'' \quad (4)$$

$$k_n w_n = E_n I_n w_n''' \quad (5)$$

$$K_n w_n' = -E_n I_n w_n'' \quad (6)$$

where w_n, E_n, and I_n are the displacement, elastic modulus, and section inertia moment of the beam at the tail end boundary, respectively.

For the i–th spring in the middle $(0 < i < n)$, the corresponding boundary continuous condition can be described as

$$\begin{cases}(w_i)_{x_i=l_i} = (w_{i+1})_{x_{i+1}=0}\\ (\frac{\partial w_i}{\partial x_i})_{x_i=l_i} = (\frac{\partial w_{i+1}}{\partial x_{i+1}})_{x_{i+1}=0}\\ (E_i I_i \frac{\partial^2 w_i}{\partial x_i^2})_{x_i=l_i} = (E_{i+1} I_{i+1} \frac{\partial^2 w_{i+1}}{\partial x_{i+1}^2})_{x_{i+1}=0}\\ (E_i I_i \frac{\partial^3 w_i}{\partial x_i^3})_{x_i=l_i} - (E_{i+1} I_{i+1} \frac{\partial^3 w_{i+1}}{\partial x_{i+1}^3})_{x_{i+1}=0} = k_i (w_i)_{x_i=l_i}\end{cases} \quad (7)$$

For n–span continuous beams, a total of $4 + 4(n - 1) = 4n$ boundary equations can be obtained. Substituting the displacement function Eq. (1) into the $4n$ boundary conditions and regarding the $4n$ coefficients of displacement function as unknown quantities to be solved, we can obtain the relation between the $4n$ coefficients and the remaining $n \times (M + 1)$ coefficients. Assuming that the terms of −2, −1, 1, and 2 of the displacement series for each span are the value to be determined, the remaining are independent variables, and the relation can be expressed as

$$\boldsymbol{A}_{iy} = \boldsymbol{C}_{\mathrm{b}} \boldsymbol{A}_{ix} \quad (8)$$

where $\boldsymbol{C}_{\mathrm{b}}$ is the relation matrix between shape function coefficients $\boldsymbol{A}_{ix}$ and $\boldsymbol{A}_{iy}$, and

$$\boldsymbol{A}_{iy} = [A_{1-2}, A_{1-1}, A_{11}, A_{12}, \dots, A_{i-2}, A_{i-1}, A_{i1}, A_{i2}, \dots, A_{n-2}, A_{n-1}, A_{n1}, A_{n2}]^{\mathrm{T}}$$

$$\boldsymbol{A}_{ix} = [A_{1-4}, A_{1-3}, A_{10}, A_{13}, A_{14}, \dots, A_{1M}, \dots, A_{i-4}, A_{i-3}, A_{i0}, A_{i3}, \dots, A_{iM}, \dots, A_{n-4}, A_{n-3}, A_{n0}, A_{n3}, A_{n4}, \dots, A_{nM}]^{\mathrm{T}}$$

At this point, the number of independent unknown coefficients remains $n \times (M + 1)$, so the displacement function constructed can satisfy the boundary condition and be expressed as

$$w_i = \sum_{m=-4,-3,0} \boldsymbol{A}_{im}\varphi_{im}(x_i) + \sum_{m=3}^{M} \boldsymbol{A}_{im}\varphi_{im}(x_i) + \sum_{m=-2,-1} \boldsymbol{C}_{\mathrm{b}}[4(i-1)+m+3]\boldsymbol{A}_{ix}\varphi_{im}(x_i) + \sum_{m=1,2} \boldsymbol{C}_{\mathrm{b}}[4(i-1)+m+2]\boldsymbol{A}_{ix}\varphi_{im}(x_i) \quad (9)$$

1.2.3 Equilibrium equation derived based on the Hamilton principle

According to the Hamilton principle, the bending equilibrium equation of continuous beam under transverse load in arbitrary boundary conditions can be deduced, that is,

$$\delta(U_{\mathrm{P}} + U_{\mathrm{S}} - W) = 0 \quad (10)$$

where

$$U_{\mathrm{P}} = \sum_{i=1}^{n} \int_0^{l_i} \frac{E_i I_i}{2} (\frac{\partial^2 w_i}{\partial x_i^2}) \mathrm{d}x_i$$

$$U_{\mathrm{S}} = \left[\frac{1}{2}k_0 w_1^2 + \frac{1}{2}K_0\left(\frac{w_1}{x_1}\right)^2\right]_{|x_1=0} + \left[\frac{1}{2}k_n w_n^2 + \frac{1}{2}K_n\left(\frac{w_n}{x_n}\right)^2\right]_{|x_n=l_n} + \sum_{i=1}^{n-1}\left(\frac{1}{2}k_i w_i^2\right)_{|x_i=l_i} \quad (12)$$

$$W = \sum_{i=1}^{n} \int_0^{l_i} q(x) w_i \mathrm{d}x_i \quad (13)$$

where U_{P} and U_{S} are the strain energy of the continuous beam and the elastic potential energy on the boundary respectively; W is external work; δ is the first variation.

Substituting Eqs. (11)–(13) into Eq. (10) and substituting the displacement expression Eq. (9) which satisfies the boundary condition into Eq. (10), we get the algebraic equations of $n \times (M+1)$ independent coefficients by the Galerkin method. Then the $n \times (M+1)$ unknown coefficients can be obtained by solving the equations. These coefficient values are substituted back into Eq. (9) to obtain the displacement under the boundary condition. According to the differential relationship of the stress, angle of rotation, section bending moment, and shear force on the section with the displacement of the beam structure, the corresponding physical quantity can be solved.

1.3 Verification of calculation examples

In order to verify the correctness of the analytical method in the paper, two boundary conditions are designed based on a three-span beam model. The boundary condition 1 simulates the situation where both ends are rigidly fixed and the intermediate support is fixed vertically, as shown in Fig. 2. The boundary condition 2 simulates the multi-span beam under elastic support. For easy calculation, the beam section shape and material properties are the same in length direction; all the span length l_i is 10 m (overall length L=30 m); elastic modulus E=210 GPa; Poisson's ratio v = 0.3. The beam section is in T shape; the web height and thickness are 400 mm and 8 mm, respectively; the panel width and thickness are 150 mm and 14 mm respectively; the rib band width and thickness are 1 000 mm and 15 mm, respectively. Assuming that the three-span beam is subjected to three sets of patch loading, and each set of patch loading is simplified into two concentrated forces, which are $F_{1,1}$ = 80 kN, $F_{1,2}$ = 120 kN; $F_{2,1}$ = 60 kN, $F_{2,2}$ = 60 kN; $F_{3,1}$ = 120 kN, $F_{3,2}$ = 80 kN. The distances D_1, D_2, and D_3 between the concentrated forces of each set of patch loading are 4, 6 and 4 m respectively. The distances X_1, X_2, and X_3 of the first concentrated force and the leftmost support of the continuous beam for each set of patch loading are 5.8, 11.8, and 20.5 m respectively.

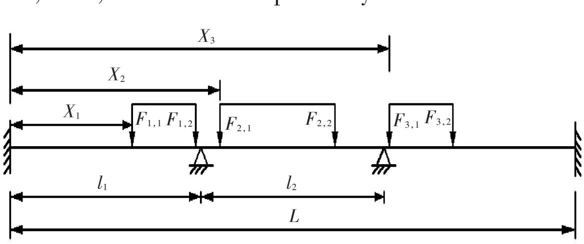


Fig.2 Schematic diagram of the three-span beams model under boundary condition 1

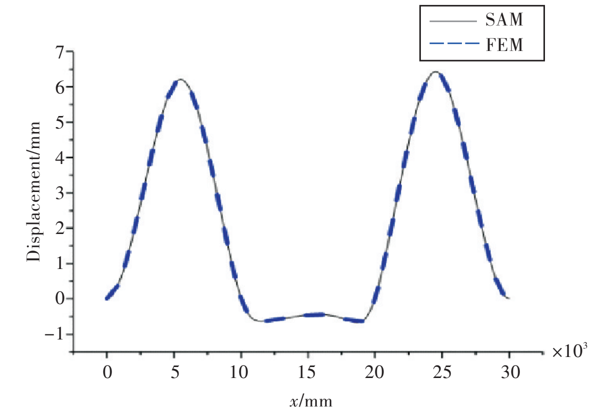
When the boundary condition 1 is calculated, the stiffness coefficients of displacement springs at both ends are taken as large values to conduct the simulation, and they are both set as $k_0 = k_n = 1 \times 10^{10}$ N/m; the stiffness coefficients of torsion springs at both ends are set as $K_0 = K_n = 1 \times 10^{10}$ (N·m)/rad. The boundary condition 2 simulates the multi-span beam under the elastic support; the stiffness coefficients of displacement springs at both ends are set as $k_0 = k_n = 1 \times 10^8$ N/m; those of torsion springs are set as $K_0 = K_n = 1 \times 10^3$ (N·m)/rad. The stiffness coefficients of intermediate supports are set as $k_1 = 5 \times 10^7$ N/m and $k_2 = 7 \times 10^7$ N/m for boundary conditions 1 and 2, respectively. Table 1 shows the influence of truncation number M on displacement results under boundary condition 1. Table 2 shows the comparison of the maximum displacement values of each span calculated by the proposed method and FEM. Fig. 3 and Fig. 4 show the comparison of the calculation results of the overall displacement, section bending moment, and section shear force of the multi-span beam under two boundary conditions with the FEM results.

Table 1 The impacts of M value on displacement under boundary condition 1

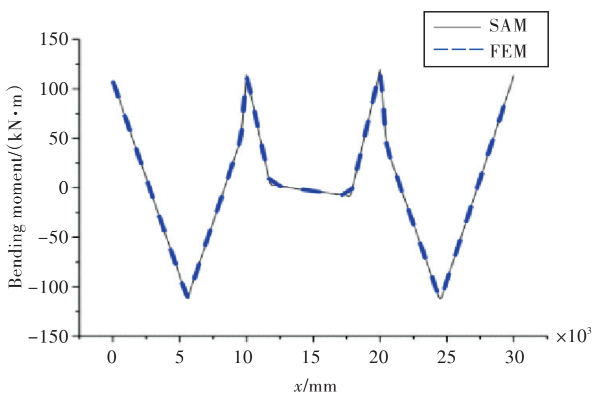
M	Displacement /mm		
	First span	Second span (minimum value)	Third span
5	6.200	−0.672	6.420
6	6.196	−0.670	6.416
7	6.195	−0.671	6.416
8	6.197	−0.671	6.418
9	6.201	−0.671	6.421
10	6.201	−0.672	6.421
11	6.201	−0.672	6.422

Table 2 The comparisons of calculated displacement for each span

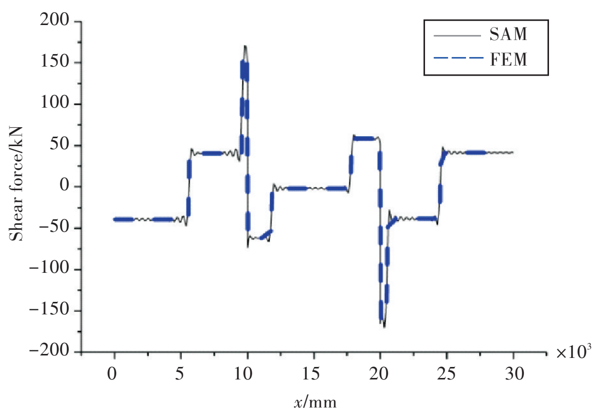
		Displacement /mm		Error/%
		Proposed method	FEM	
Fixed support boundary	Maximum displacement of the first span	6.203	6.201	<0.05
	Maximum displacement of the second span	−0.673	−0.674	<0.05
	Maximum displacement of the third span	6.424	6.421	<0.05
Elastic support boundary	Maximum displacement of the first span	15.206	15.204	<0.05
	Maximum displacement of the second span	0.896	0.895	<0.05
	Maximum displacement of the third span	14.417	14.415	<0.05



(a) Displacement

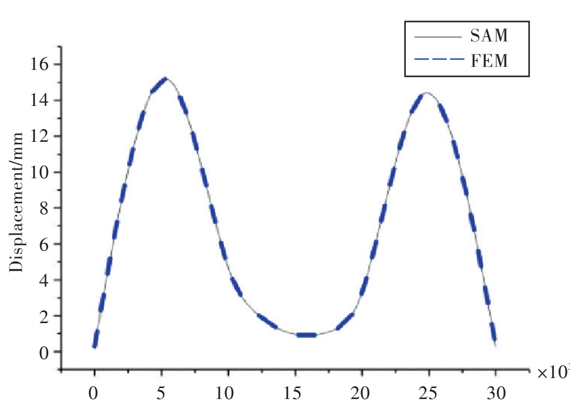


(b) Bending moment

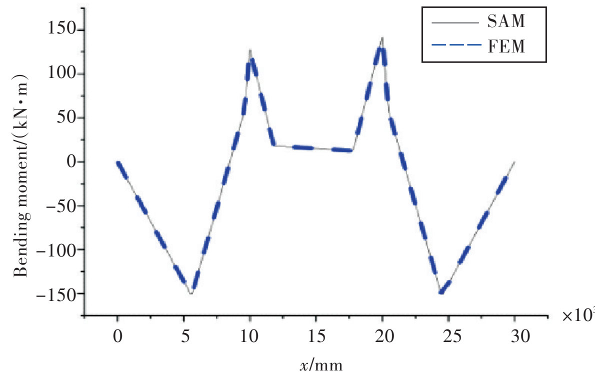


(c) Shear force

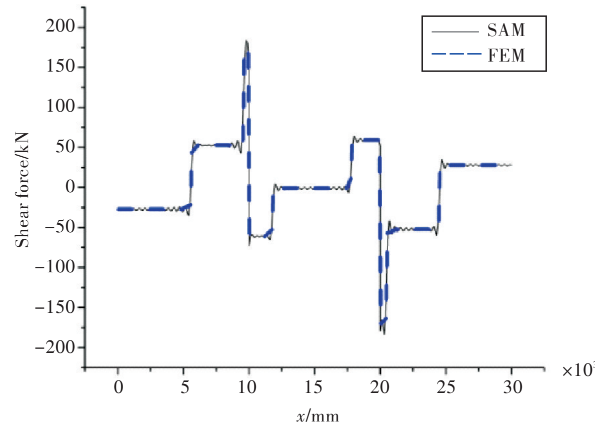
Fig.3 Result comparisons of the proposed method and FEM under boundary condition 1



(a) Displacement



(b) Bending moment



(c) Shear force

Fig.4 Result comparisons of the proposed method and FEM under boundary condition 2

As can be seen from Table 1, when the truncation number M=10, the whole calculation tends to be stable, and the convergence of the calculation results is good. As can be seen from Table 2, the error of the two methods is small.

Fig. 3 and Fig. 4 show that the shear force is a series of horizontal lines, and the shape functions in the form of sines and cosines selected in the paper need more terms for the horizontal line simulation. After comprehensive consideration, the truncation number is set as M= 40. On the basis of the calculation results in Table 2, Fig. 3 and Fig. 4, it can be seen that the overall displacement, section bending moment, and section shear force calculated by the proposed method basically coincide with the results calculated by FEM, which verifies the correctness of the proposed method. At the same time, compared with the FEM, the proposed method is convenient in calculation and easy to be set up, so it has good engineering application value.

2 Worst-case analysis of multi-span beams under patch loading

2.1 Physical model

The worst-case analysis of multi-span beams under patch loading can be described as follows [8]: Given the geometric structure of multi-span beams, the number of patch loading sets and the size of patch loading, the worst-case under the patch loading is found, even if the section bending moment, section shear force, or overall displacement of a span beam reaches the maximum value. Due to a large amount of collocation of patch loading layout, it is difficult to directly determine the corresponding worst-case location when the maximum internal force is generated by multi-span beams, so the genetic algorithm is used to optimize the calculation of the problem.

Fig. 5 shows the physical model of multi-span beams under patch loading. Assuming that e sets of patch loading act on multi-span beams, the number of patch loading in the j-th set of patch loading is s_j; $F_{j,a}$ is the size of the a-th patch loading in the j-th set of patch loading; X_j is the distance between the first load in the j-th set of patch loading and the left end of the multi-span beam; $D_{j,a}$ is the distance between the a-th patch and the $(a+1)$-th patch in the j-th set of load patch loading.

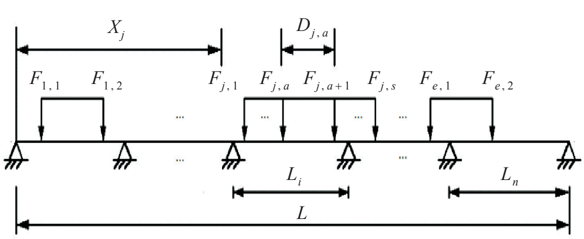


Fig.5 Schematic diagram of the multi-span beams model under multiple patch loading [8]

2.2 Mathematical model

2.2.1 Design variables

When the worst-case of multi-span beams under patch loading is analyzed, the variable is the location of each patch loading. Corresponding to the above physical model, when the location of the first load in a set of patch loading is determined, the location of the set of patch loading is determined correspondingly. Therefore, the distance X_j between the first load of each set of the patch and the left end of multi-span beams is taken as the design variables of the problem.

2.2.2 Objective function

In the worst-case of multi-span beams under the patch loading, when the maximum bending moment M_i, the maximum shear force F_{Si}, or the maximum deformation w_{max} of a span on a multi-span beam is taken as the objective function, the objective function value of each load case is solved by the SAM derived in this paper. The results show that it has good accuracy and greatly reduces the calculation time compared with the FEM.

2.2.3 Constraint conditions

On the loading deck of a real ship, the distance between adjacent vehicles needs to meet certain requirements. Therefore, the constraint condition for the worst-case analysis of multi-span beams under patch loading is the distance between the two adjacent sets of patch loading. The constraint conditions are shown in Eq. (14), where $Con1_j$ and $Con2_j$ are the minimum and maximum distances to be satisfied between the j-th set and the $(j+1)$-th set, respectively.

$$\begin{cases} X_{j+1}-(X_j+\sum_{a=1}^{s-1}D_{j,a})\geqslant Con1_j \\ X_{j+1}-(X_j+\sum_{a=1}^{s-1}D_{j,a})\leqslant Con2_j \end{cases} \tag{14}$$

2.3 Calculation examples

The proposed bending calculation method of multi-span beams and the genetic algorithm are used to solve the above optimization problem. For the convenience of comparison, scheme 2 in Reference [8] is selected for calculation. The geometric parameters and section parameters, as well as the quantity, size, and distances of each set of patch loading in this scheme are the same as the three-span beams used in Section 2.1. However, the distances between the first load of each set and the left end of the multi-span beam, X_1, X_2, and X_3, are design variables to be solved, whose value ranges are 0.5–6 m, 10–14 m and 20–25.5 m, respectively. The constraint conditions are that the minimum distance between the two adjacent sets of patch loading is no less than 1 m, and the maximum distance is no more than 5 m. The constraint conditions used in the boundary are rigidly fixed at both ends and vertically fixed at the intermediate support. In other words, the stiffness coefficient of the displacement spring at both ends is set as 1×10^{10} N/m; that of the torsion spring at both ends is set as 1×10^{10} (N·m)/rad; that of the displacement spring in the intermediate support is set as 1×10^{10} N/m. The results of the worst-case are obtained by ga function of the genetic algorithm in Matlab software, which can deal with the optimization problem of the continuous value of design variables. In the calculation, the population

Table 3 Numerical results of the worst-case analysis of multi-span beams

Objective function	Maximum bending moment/(kN·m) or shear force/kN		Values of design variables in the worst-case/m					
	Proposed method	Method in Reference[8]	Proposed method			Method in Reference[8]		
			X_1	X_2	X_3	X_1	X_2	X_3
Maximum bending moment of the first span	262.07	253.91	1.87	10.00	21.00	2	10	21
Maximum bending moment of the second span	157.12	153.71	6.00	11.84	22.35	6	12	22.5
Maximum bending moment of the third span	262.07	253.91	5.00	14.00	24.13	5	14	24
Maximum shear force of the first span	163.60	159.30	5.84	10.87	20.68	5.5	12	20
Maximum shear force of the second span	86.43	84.39	6.00	13.78	22.51	6	13.5	22.5
Maximum shear force of the third span	163.60	159.30	5.32	13.13	20.16	6	12	20.5

number is set as 100, and the generations are set as 100. The comparison between the final optimization results and the numerical results in Reference [8] is shown in Table 3.

As can be seen from Table 3, compared with Reference [8], the genetic algorithm adopted in this paper can deal with the optimization problem of the continuous value of design variables, so it can find a more accurate worst-case of patch loading. The maximum bending moment or maximum shear force in this worst-case is larger than that in Reference [8]. Meanwhile, the continuous value of design variables is closer to engineering practice, because the movement of vehicles on the loading deck is continuous rather than in a step change.

3 Conclusions

Based on the energy principle and boundary conditions, the bending problem of continuous multi-span beams under arbitrary boundary conditions is analyzed. By comparing the results of the proposed method with the results of FEM, we verify the correctness of the proposed method. The method is used to analyze the worst-case of multi-span beams under patch loading, and the satisfactory results are obtained with high calculation accuracy and less time consumption. The main conclusions are as follows:

1) With the increase in the truncation number of Fourier series, the results of the deduced calculation method converge quickly and the numerical stability is good. In comparison with the example results of FEM, the displacement error is less than 0.05%. At the same time, since the calculation process in the paper based on the energy principle does not need an equilibrium equation, it is easy to generalize to plate structure and other more complex structures.

2) When calculating and analyzing the worst-case of multi-span beams under patch loading, the proposed method obtains a more accurate worst-case.

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离心风机流场大涡模拟与管中噪声数值预报

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摘　要：［**目的**］为开展离心风机振动噪声预报，［**方法**］首先，对离心风机整个流域进行结构化网格划分，采用大涡模拟(LES)获得叶轮表面随时间变化的脉动力，以旋转偶极子辐射为模型，采用声学有限元方法计算管道中的辐射声；然后，以蜗壳部分的壁面偶极子为辐射源，计算壁面脉动压力辐射声；最后，将数值计算结果与试验结果进行对比分析。［**结果**］结果表明，蜗壳壁面脉动压力辐射声在低频段高于叶轮辐射声，而在高频段低于叶轮辐射声；数值计算结果与实验结果具有一致性。［**结论**］所提叶轮辐射声及蜗壳壁面脉动压力辐射声的预报方法可满足工程应用需求。

关键词：离心风机；管中噪声；大涡模拟；旋转偶极子辐射模型；声学有限元法

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任意边界下多跨梁弯曲计算及其工程应用

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摘　要：［**目的**］为了快速寻找到甲板结构在轮印载荷下的最危险工况，［**方法**］针对任意边界条件下多跨梁弯曲问题，首先利用改进的傅里叶级数方法(IFSM)描述多跨梁的位移函数，列出位移函数需满足的边界方程，并求解得到级数中各系数间的关系式；然后，基于哈密顿原理得到能量控制方程，采用伽辽金方法求解出满足边界条件的梁结构位移函数，通过算例，与有限元结果进行对比，验证此方法的正确性；最后，将该方法应用于轮印载荷下多跨梁最危险工况的计算中。［**结果**］结果表明，所用方法的计算结果与有限元结果的误差小于0.05%，具有很好的精度。［**结论**］相比有限元法，所用方法求解多跨梁最危险工况的速度得到极大提高，同时结合遗传算法，可获得更为精确的轮印载荷最危险工况的作用位置。

关键词：多跨梁；轮印载荷；弯曲；任意边界条件；哈密顿原理；最危险工况分析