To cite this article: LIN X H, YANG D P, GONG C L, et al. Aircraft maximum density layout algorithm based on multi-constraint two-dimensional packing [J/OL]. Chinese Journal of Ship Research, 2021, 16(6). http://www.ship-research.com/EN/Y2021/V16/I6/27.

DOI: 10.19693/j.issn.1673-3185. 02022

Aircraft maximum density spotting algorithm based on multi-constraint two-dimensional packing

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Abstract: [**Objectives**] A multi-constraint two-dimensional packing algorithm is used to determine carrier-based aircraft's maximum density spot factor. [**Methods**] First, the constraints are presented. Then, based on a lowest-gravity-center NFP algorithm combined with mathematic modeling of the distance constraints and a heuristic algorithm for the "keep-to-the-boundary" positioning, algorithms for maximum density spotting on the flight deck and in the hangar are presented. [**Results**] With these algorithms, the maximum spotting numbers of F/A-18C and F-35C aircraft, respectively, on a Nimitz-class aircraft carrier are determined, as well as the spot factor of F-35C aircraft, and the results are consistent with the known facts. [**Conclusions**] With these algorithms, the spot factor of carrier-based aircraft can be quickly calculated, making them useful for guiding the spotting suitability design of newly-built carrier-based aircraft.

Key words: carrier-based aircraft; maximum density spotting; multi-constraint; two-dimensional packing; boundary overlapping

CLC number: U662.2

0 Introduction

Carrier-based aircraft are the main combat force of aircraft carriers (carriers for short hereinafter), so accommodating more carrier-based aircraft is important for improving combat effectiveness [1]. Considering the limited deck space of carriers, researchers need to miniaturize carrier-based aircraft and optimize their contours without compromising combat performance to increase the number of carrier-based aircraft a carrier can base and improve the carrier suitability of the aircraft [2]. To quantitatively assess the footprint of a carrier-based aircraft on the flight deck and hangar, the U. S. Navy proposed a "maximum density spot factor" according to their experience in carrier-based aircraft utilization for about 100 years [3-4].

The maximum density spot factor, or spot factor for short, is an empirical indicator that effectively characterizes the amount of flight deck or hangar space required for basing a given carrier-based aircraft. A large spot factor indicates a larger area the carrier-based aircraft occupies. The spot factor of a carrier-based aircraft is calculated by the following process [5].

1) Set F/A-18C as the baseline carrier-based aircraft and specify its spot factor as 1.

2) Fill the flight deck (excluding the landing area) and hangar of the carrier Nimitz with the baseline carrier-based aircraft according to given rules and constraints, and record the total number of carrier-based aircraft on the flight deck and in the han-

Received: 2020－07－03 **Accepted**: 2020－12－08

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gar, which is then set to *a*.

3) Place a certain type of carrier-based aircraft according to the same rules, record the total number of the carrier-based aircraft placed, and set it to *b*.

4) The ratio of the number of baseline carrier-based aircraft to that of the carrier-based aircraft under investigation, namely *a*/*b*, is the spot factor of the carrier-based aircraft under investigation.

The maximum density spotting of carrier-based aircraft is the core in calculating the spot factor, and it is generally implemented manually by experienced staff with the help of CAD both at home and abroad. The design work consumes much time and manpower, making the calculation of the spot factor inefficient and in urgent need of an algorithm that can quickly and automatically generate a plan for a maximum density spotting of carrier-based aircraft.

The maximum density spotting of carrier-based aircraft can be classified as a problem of packing two-dimensional irregular objects under multiple constraints [6]. As the problem of packing two-dimensional irregular objects had a large solution space, early researchers mainly transformed it into a simpler problem of packing regular polygon objects by jointing and combining [7-8]. However, this method would waste much space when it is applied to the packing of objects with extremely irregular shapes. Thus, Adamowicz et al. [9] proposed the no-fit polygon (NFP) algorithm on the basis of polygon representation, and they determined the overlap between polygons by calculating the NFPs of irregular polygons. Zhang [10] designed an automatic spot software for carrier-based aircraft by the gravity-center NFP algorithm but failed to consider the multiple constraints on the maximum density spotting of carrier-based aircraft.

For the above reasons, we will focus on the mathematical expressions, processing strategies, and basic calculations regarding the maximum density spotting of carrier-based aircraft under multiple constraints on the basis of the two-dimensional packing algorithm and propose algorithms for the maximum density spotting on the flight deck and in the hangar respectively.

1 Overview of constraints on maximum density spotting of carrier-based aircraft

Generally, compared with the conventional two-dimensional packing, the maximum density spotting of carrier-based aircraft on the flight deck and in the hangar must satisfy the following constraints.

1) Distance constraints. Safe distances should be kept between carrier-based aircraft and between the carrier-based aircraft and the carrier boundaries/hull structures to meet the spot safety requirements. Among them, the distances between the carrier-based aircraft and the carrier boundary/hull structure include the distances between the carrier-based aircraft and the outer boundaries of the carrier (such as the deck boundary and the flight line), the distances between the carrier-based aircraft and hull structures (such as the island superstructure, the hangar wall, and the fire-compartment hangar door). As carrier-based aircraft will be transferred, the distance between the carrier-based aircraft is different from that between the carrier-based aircraft and the island superstructure.

2) Boundary-overlapping constraints. The carrier-based aircraft fuselage should maintain a distance from the hull structure and the flight line on the flight deck, and they should not overlap with each other. However, when a carrier-based aircraft is placed close to the side of the flight deck, the carrier-based aircraft fuselage can exceed the deck edge on the premise of a safe distance between the carrier-based aircraft wheels and the deck edge. In other words, the contour boundary of the carrier-based aircraft should be considered dynamically during spot.

2 Mathematical expressions of maximum density spotting of carrier-based aircraft

2.1 Geometric expressions of carrier-based aircraft, flight deck, and hangar

The contours of the carrier-based aircraft, flight deck, and hangar are all irregular shapes. Their geometric representation involves storage, intersection, translation, rotation, and other image operations, which directly influence the time consumption and accuracy of the spotting algorithm. Currently, two-dimensional irregular shapes are mainly represented by original curve representation, polygon representation, and the envelope method. This paper employs polygon representation for geometric expressions of the carrier-based aircraft, flight deck, and hangar.

In polygon representation, connected polygons are used to represent the shapes of the deck and carrier-based aircraft in the spot. If the contours of the carrier-based aircraft and the deck have curves, the curves are replaced approximately with line segments to certain accuracy (Fig. 1).

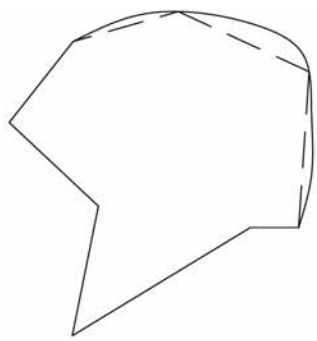


Fig. 1 Polygon representation

A connected polygon is defined as follows.

Definition 1: A planarly connected polygon *P* with *N* sides refers to an orderly set of *N* vector endpoints. These *N* vector endpoints are connected end to end and are arranged in the counterclockwise (or clockwise) direction, and no interior point intersection between vectors occurs.

It can be represented mathematically as $P=\{P_1(X_1, Y_1), P_2(X_2, Y_2), \ldots, P_i(X_i, Y_i), \ldots P_n(X_N, Y_N), i=1, \ldots N\}$.

Polygon representation reduces the calculation amount between graphs and improves the packing speed. Although it decreases the accuracy of irregular shapes to a certain extent, it can satisfy the requirement for spot accuracy as long as the discrete accuracy is high enough.

2.2 Mathematical model of distance constraints and processing strategy

As shown in Fig. 2, there are three distance constraints, where l_1 is the distance between two carrier-based aircraft; l_2 is the distance between the carrier-based aircraft and an outer boundary of the carrier; l_3 is the distance between the carrier-based aircraft and an inner boundary of the carrier such as the island superstructure.

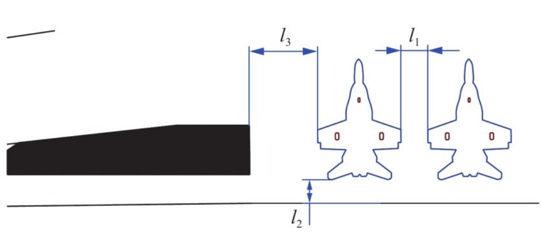


Fig. 2 Distance constraints on two-dimensional packing

In this paper, we use offset scaling to process the distance constraints. Specifically, we scale up or down the boundaries of the carrier-based aircraft and the deck at certain spacings and then carry out the maximum density spotting. The actual distance between two carrier-based aircraft is set to l_1', and the actual distances between a carrier-based aircraft and the inner and outer boundaries of the carrier are respectively set to l_3' and l_2', as shown in Equation (1). Then, the carrier-based aircraft boundary is scaled up according to the distance l_1'; the outer contour of the deck is offset-scaled down according to l_2', and the inner contour of the deck is scaled up according to l_3'.

$$\begin{cases} l_1' = 0.5 l_1 \\ l_2' = \begin{cases} l_2 - l_1', & l_2 > l_1' \\ 0, & l_2 < l_1' \end{cases} \\ l_3' = \begin{cases} l_3 - l_1', & l_3 > l_1' \\ 0, & l_3 < l_1' \end{cases} \end{cases} \tag{1}$$

Then, we conduct offset operations on the carrier-based aircraft and the carrier by the pair-wise offset method in the following steps.

Step 1: Conduct the offset operation on each line segment of the boundaries of the carrier-based aircraft and the carrier in the offset direction. As shown in Fig. 3, the starting and ending points of the line segment are $P_s(x_s, y_s)$ and $P_e(x_e, y_e)$ respectively, $\boldsymbol{n}$ is a normal vector, and $\boldsymbol{d}$ is the spacing. After offset treatment, the starting and ending points change to P_s' and P_e', and $\boldsymbol{P}_{se}$ is the directed line segment from P_s to P_e. Then

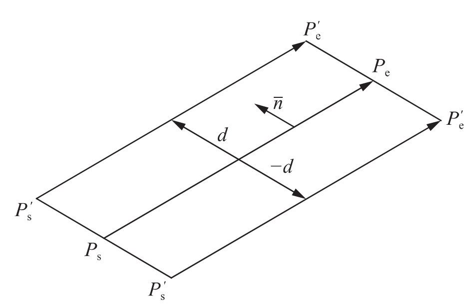
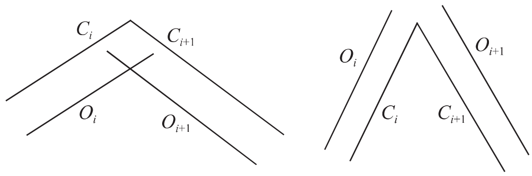


Fig. 3 Offset calculation for lines

$$\begin{cases} \boldsymbol{n} = \left(\dfrac{y_e - y_s}{|\boldsymbol{P}_{se}|} - \dfrac{x_e - x_s}{|\boldsymbol{P}_{se}|} \right) \\ P_s' = P_s \pm d \times \boldsymbol{n} \\ P_e' = P_e \pm d \times \boldsymbol{n} \end{cases} \tag{2}$$

Step 2: Close the boundary obtained by offset operation. The adjacent two sides C_i and C_{i+1} of the polygon after offset operation are clipped if they intersect and are connected if they are separated, as shown in Fig. 4.

In this paper, we connect two separated line segments by extending them (Fig. 5).



(a) Intersecting after offset (b) Separated after offset

Fig. 4 Geometric relation of line segments after offsetting operation

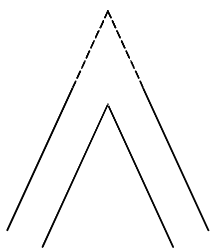


Fig. 5 Connecting operation between line segments

Step 3: Check the original offset boundary generated and remove the invalid boundary to obtain the final valid boundary.

2.3 Mathematical model of boundary-overlapping constraints and processing strategy

The packing on the flight deck is subject to boundary-overlapping constraints. The two-dimensional boundary-overlapping packing problem can be represented as follows. A set of two-dimensional irregularly shaped parts P_1, P_2, …, P_n, with each part containing feature points $\{D_1, D_2, \dots, D_m\}$. The parts P_1, P_2, …, P_n should be placed on the template P in the maximum density spotting under the following two constraints:

1) Two arbitrary parts P_i and P_j do not overlap each other.

2) The feature points $\{D_1, D_2, \dots, D_m\}$ of the part P_i keep a distance to the outer boundary of the template P.

Fig. 6 presents a boundary-overlapping spot of carrier-based aircraft. In other words, the carrier-based aircraft can be placed with the fuselage boundary exceeding the boundary of the flight deck as long as the wheels (usually one front wheel and two main wheels) and other feature points of the carrier-based aircraft do not extend beyond the boundary of the flight deck.

The conventional packing algorithm cannot be used to calculate the boundary-overlapping packing problem. The template can be divided into the boundary-overlapping packing area at the edge of the flight deck and the conventional packing area in

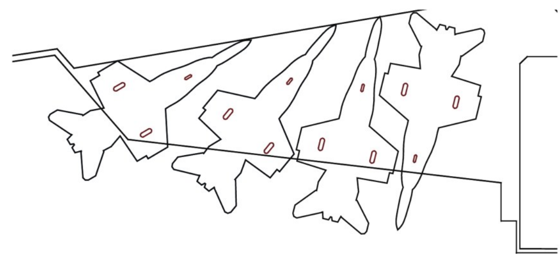


Fig. 6 Carrier-based aircraft spot plan under boundary-overlapping constraints

the inner area. The packing problems in the two areas can be solved by different algorithms in order. If the carrier-based aircraft are placed at the edge firstly and then the inner area of the deck, the spot is in the order of shrinking from the outside. If the carrier-based aircraft are placed in the inner area and then the edge of the deck, the spot follows the order of expanding from the inside (Fig. 7).

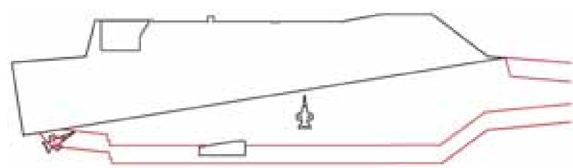


Fig. 7 Different regions of flight deck

The order of packing in an inward shrinking manner we adopt can be summarized as follows: On the premise of the aircraft wheels not exceeding the deck boundary, we fill the outer boundary of the flight deck with carrier-based aircraft successively in a way of moving as many carrier-based aircraft as possible toward the deck boundary and the lowest-gravity-center position. Then, we place carrier-based aircraft in the inner area of the deck.

To sum up, the positioning steps of the boundary-overlapping spot are as follows.

1) Import the contour boundary information of the carrier-based aircraft and the flight deck in a certain graphic format, and identify the feature points of the carrier-based aircraft with different layers in the graph.

2) Obtain the feature points of the carrier-based aircraft, and determine whether the carrier-based aircraft exceeds the deck boundary according to whether the feature points are beyond the deck area.

3) Perform the packing in an inward shrinking order, namely that we start with the carrier-based aircraft spot at the edge of the flight deck and conduct the spot in the inner area subsequently.

3 Algorithm for carrier-based aircraft packing in the hangar

The maximum density spotting of carrier-based aircraft in the hangar can be considered as two-dimensional packing of irregular objects under non-offset constraints, and the optimization objective is to maximize the number of carrier-based aircraft in the hangar. We will solve this problem by the lowest-gravity-center NFP algorithm [11], and the algorithm flow chart for the maximum density spotting of carrier-based aircraft in the hangar is shown in Fig. 8.

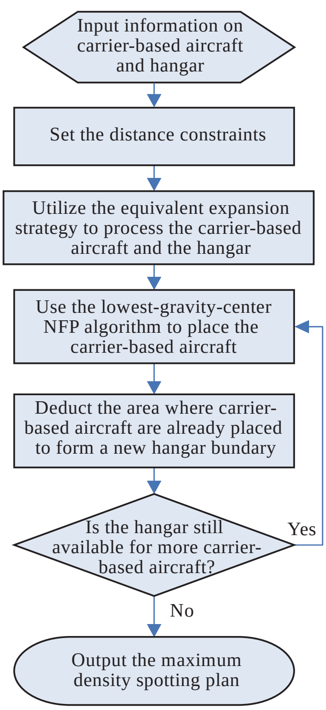


Fig. 8 Flow chart for maximum density spotting in the hangar

The algorithm is carried out in the following steps.

1) Import the contour boundary information of the carrier-based aircraft and the hangar in the .dxf format, and convert them into discrete points by polygon representation.

2) Input the distance between carrier-based aircraft, that between the carrier-based aircraft and the hangar comparment door, and that between the carrier-based aircraft and the hangar wall according to the requirement of the maximum density spotting rule.

3) Use the equivalent expansion strategy to process the boundaries of the carrier-based aircraft and the hangar on the basis of Equations (1) and (2).

4) Place carrier-based aircraft with the lowest-gravity-center NFP algorithm. According to the packing rule of this algorithm, a carrier-based aircraft is placed in the lowest-gravity-center position. This algorithm intends to place carrier-based aircraft to the lowest center of gravity on the premise of no boundary overlapping and thereby leave more space for carrier-based aircraft to be placed later. This algorithm is performed in the following steps: The center of gravity of the carrier-based aircraft is obtained from its contour boundary; the aircraft is rotated at a specified angle interval, which is 1° in this paper; the gravity-center NFP under each rotation angle is solved; the lowest-gravity-center position and the corresponding rotation angle are recorded for carrier-based aircraft positioning. A schematic diagram of the lowest center of gravity can be obtained by the lowest-gravity-center NFP method (Fig. 9).

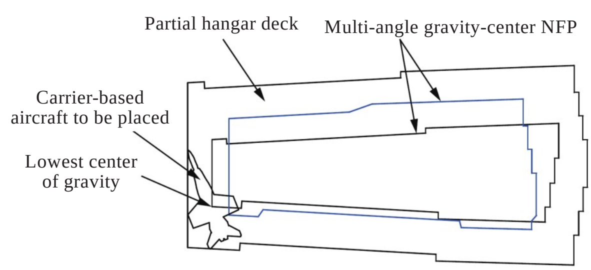


Fig. 9 Lowest-gravity-center NFP algorithm

5) Deduct the area where carrier-based aircraft are already placed from the hangar deck to obtain the latest hangar contour (Fig. 10), and continue carrier-based aircraft spot.

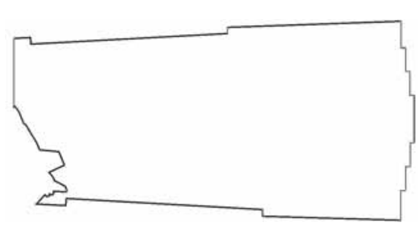


Fig. 10 Generation of a new hangar contour

6) Repeat the above steps until the remaining hangar space is not enough for one more carrier-based aircraft.

4 Algorithm for carrier-based aircraft packing on flight deck

To obtain the maximum density spotting of carrier-based aircraft on the flight deck, we respectively calculate the maximum density spotting plans for carrier-based aircraft at the edge and those in the inner area of the flight deck in the inward shrinking order [12]. For this purpose, we use the heuristic algorithm for keep-to-the-boundary positioning to place the carrier-based aircraft at the edge of the flight deck until the edge area cannot accommodate more carrier-based aircraft under the boundary-overlapping constraints. Then, we use the NFP algorithm, which is used for the maximum density spotting of carrier-based aircraft in the hangar, to pack carrier-based aircraft in the inner area of the flight deck until the inside area is fully occupied. The algorithm flow for carrier-based aircraft packing on the flight deck is shown in Fig. 11.

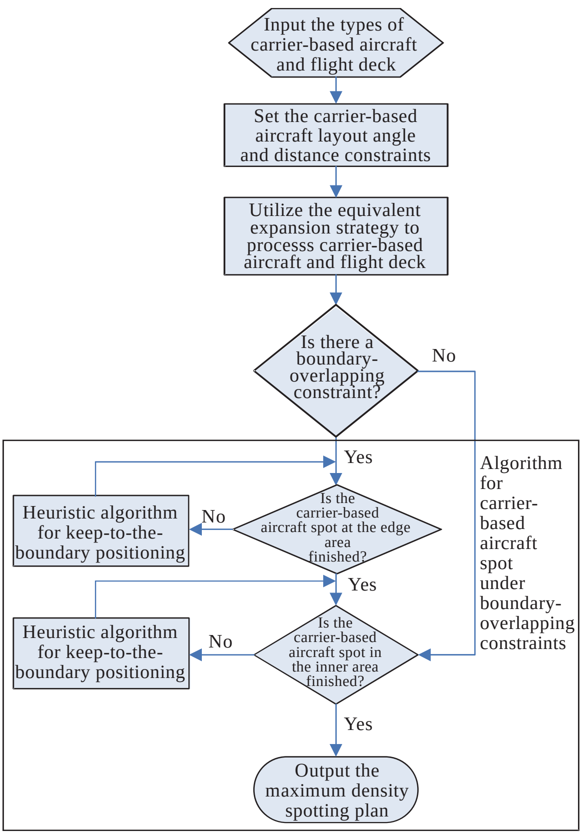


Fig. 11 Algorithm flow for carrier-based aircraft spot under boundary-overlapping constraints

The main idea of the heuristic algorithm for keep-to-the-boundary positioning is as follows: For the edge L_1 (Fig. 12), the leftmost and lowermost position of the deck is used as the local optimal position for a part. We use this strategy to place the carrier-based aircraft one by one along the opposite direction of L_1 until no more carrier-based aircraft can be placed in this direction. Then, we place the remaining carrier-based aircraft along the opposite directions of L_2–L_4 successively in a similar way, except that the local optimal positions of a part are updated to the rightmost and lowermost position, the rightmost and uppermost position, and the leftmost and uppermost position respectively. In this way, the carrier-based aircraft spot at the edge of the flight deck is completed.

The maximum density spotting of carrier-based aircraft at the edge of the flight deck with this algorithm is performed in the following steps (Fig. 12).

1) Starting from the rightmost position in the stern of the flight deck, we place carrier-based aircraft along the right deck edge in the direction from the stern to the bow with the rightmost corner of the stern as the local optimal position of a part until we cannot place more carrier-based aircraft in this direction.

2) In a similar way, we place carrier-based aircraft along the bow deck edge and the left side edge respectively with the rightmost corner and the leftmost corner of the bow as the local optimal positions of a part.

5 Experimental results of the algorithms

We used the above algorithms to perform a maximum density spotting of F/A-18C aircraft (Fig. 13). The sizes and boundaries of the flight deck and the hangar were determined in light of Reference [13]. We placed 76 and 53 aircraft, a total of 129 aircraft, on the flight deck and in the hangar respectively. This result is consistent with the maximum density spotting of 130 F/A-18C aircraft on the flight deck and in the hangar published by the U.S. Army [14].

With the F/A-18C aircraft as the baseline carrier-

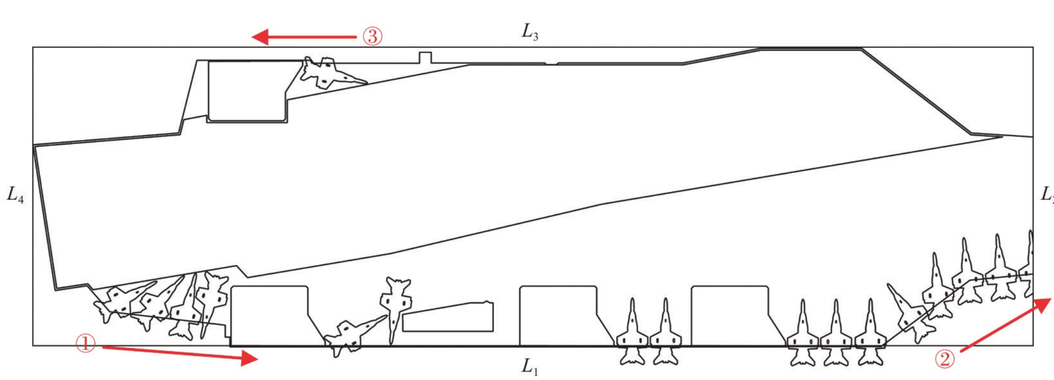


Fig. 12 Flow of heuristic algorithm for keep-to-the-boundary positioning

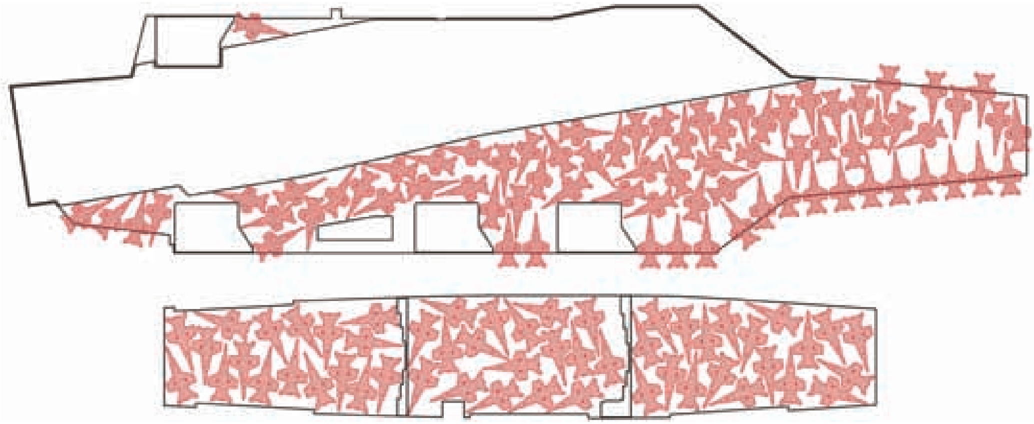


Fig. 13 Maximum spot pattern of the F/A-18C aircraft on the flight deck and in the hangar

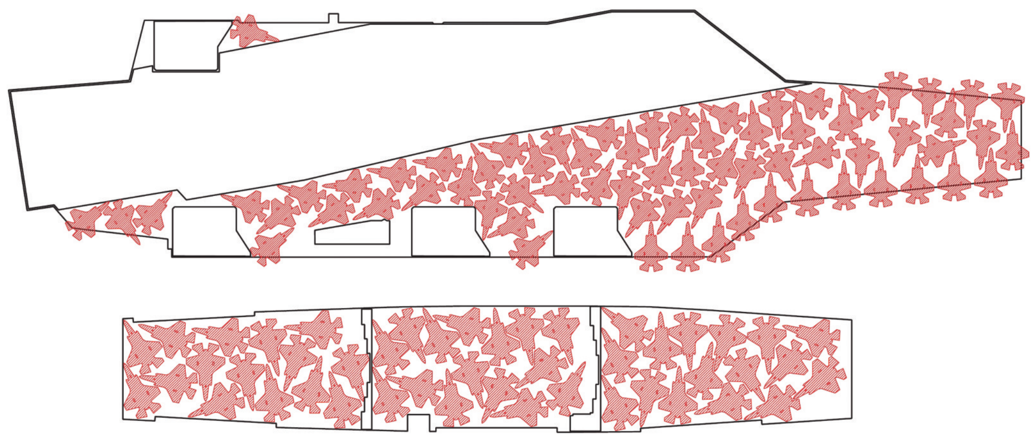


Fig. 14 Maximum spot pattern of the F-35C aircraft on the flight deck and in the hangar

based aircraft, we carried out a maximum density spotting of F-35C aircraft (Fig. 14). We placed 69 and 46 aircraft, respectively, on the flight deck and in the hangar, namely that 115 F-35C aircraft can be placed at most. According to the definition, its spot factor is 1.12, which is comparable to the value 1.11 published by the U.S. Army [4].

6 Conclusion

Resorting to the lowest-gravity-center NFP algorithm, we analyzed the distance constraints and the boundary-overlapping constraints on the maximum density spotting of carrier-based aircraft. We proposed a mathematical model of the distance constraints and the processing strategy and adopted the heuristic algorithm for keep-to-the-boundary positioning to process the boundary-overlapping constraints. Then, we provided algorithms for maximum density spotting of carrier-based aircraft on the flight deck and in the hangar, respectively, under their specific constraints. With the two algorithms, we concluded that the carrier Nimitz could accommodate 129 F/A-18C aircraft or 115 F-35C aircraft at most and that the spot factor of the F-35C aircraft was 1.12. Basically consistent with the data published by the U.S. Army, these results verify the practicability of the proposed algorithms.

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基于多约束二维排样的舰载机最大密度布列算法

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摘　要：[**目的**]采用多约束条件下的二维排样算法解决舰载机最大密度布列问题。[**方法**]梳理出最大密度布列的约束条件，然后以最低重心 NFP 算法为基础，提出距离约束的数学模型及处理策略，并利用启发式靠边定位算法来处理超边界约束，最后分别形成飞行甲板和机库最大密度布列的算法。[**结果**]利用该算法得到"尼米兹"航母可最大密度布列 F/A-18C 或 F-35C 飞机的数量，以及 F -35C 飞机的布列因子，与美军相关文献公布的舰载机布列数据基本吻合。[**结论**]利用该算法可快速得出舰载机的布列因子，可指导新研舰载机的布列适配性设计。

关键词：舰载机；最大密度布列；多约束；二维排样；超边界